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INTEGRATING GRAPHICAL AND ANALYTICAL APPROACHES IN THE STUDY OF MOTION IN MECHANICS

ІНТЕГРАЦІЯ ГРАФІЧНИХ ТА АНАЛІТИЧНИХ ПІДХОДІВ У ВИВЧЕННІ РУХУ В МЕХАНІЦІ

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ABSTRACT

When studying the kinematics of point motion, students often encounter difficulties in understanding the basic concepts of «displacement» and «path». This article proposes an effective approach to studying these important concepts through the detailed analysis of a one-dimensional problem. To facilitate the learning process, two solution methods are used: geometric and analytical. Comparing these approaches allows students to understand their characteristics and choose the most effective method for specific problem conditions.

The main focus of the paper is on calculating various motion parameters, such as velocity, stopping time, displacement, and path length, using the Wolfram Mathematica software environment. This software provides students with powerful analytical tools, allowing them to perform complex calculations, analyze results, and simulate various motion scenarios. Students can easily change parameters like initial speed, acceleration, or duration of movement and observe the resulting changes. This process enables a deeper understanding of physical phenomena.

After conducting an analytical study, where students calculate motion parameters, using Excel to create graphs makes the learning process more interactive and visual. Excel's capabilities allow students to easily visualize data through graphs that illustrate changes in motion parameters over time. This approach not only simplifies the learning of complex concepts but also promotes skill development in modern technologies for data analysis and modelling of physical processes, providing comprehensive learning that integrates both theory and practice.

Thus, integrating computer technologies into the educational process enhances the quality of education, engages students, and prepares them to solve real-world problems in scientific and practical research.

Key words: *kinematics, path, motion, stopping point, analytical and graphical method.*

Introduction. This scientific study focuses on assessing the impact of contemporary global informatization on physics education. The integration of computer programs such as Wolfram Mathematica and Excel opens new avenues for mastering physical concepts. Our focus is on modeling the motion of a point particle, exemplified through the case of uniform one-dimensional motion [1, 1159]. However, mastering this model can be challenging, requiring an understanding of mathematical concepts and the analysis of graphical dependencies from a physical perspective. The objective of this study is to examine and compare two approaches to mathematically model mechanical motion, aiming to enhance the learning process and foster a deeper understanding of physical processes.

Methods and research methodologies. The primary approaches under consideration are the geometric and analytical methods [2, 85]. The geometric approach enables a visual understanding of body movement through graphs and diagrams [3, 14657.7], whereas the analytical approach [4, 54] requires a more profound formal comprehension of mathematical dependencies and the use of computational tools. The integration of these two methods not only facilitates a mathematical analysis of body movement but also provides a visual observation of its dynamics and changes over time.

Results and discussions

1. Analytical Approach to Determining Characteristics of Point Motion

In this section, we will delve deeper into the motion of a point particle using the example of one-dimensional motion [5, 14], defined by the expression

$$x(t) = t^2 - 10t + 3\ , \qquad (1)$$

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where $x(t)$ represents the point's coordinate at time t. We will assume that t lies within the intervals of 1 to 8 seconds.

Equation (1) represents a quadratic function, indicating the parabolic nature of the motion trajectory in configuration space. We will employ an analytical approach by calculating the derivative of Equation (1). Considering the direction of movement, we ascertain the sign of this derivative at specific moments in time. We find that

$$v(t) = \frac{dx}{\mathrm{dt}} = 2t - 10\ . \qquad (2)$$

By substituting the time values $t = 1$ s and $t = 8$ s into formula (2), we obtain

$$v(1) = 2 \cdot 1 - 10 = -2\,\mathrm{m/s}\ ,$$

$$v(8) = 2 \cdot 8 - 10 = 6\,\mathrm{m/s}\ . \qquad (3)$$

Upon analyzing the results, we conclude that at the moment of time $t = 1$ s, the derivative's value is negative, indicating that the speed of the point's movement is directed to the left (against the increase in the value of x).

At the moment of time $t = 8$ s, the derivative's value is positive, signifying that the speed of the point's movement is directed to the right (in the direction of an increasing value of x). Thus, in the time interval from 1 s to 8 s, the point particle undergoes a change in the direction of its movement, indicating the presence of a stopping point.

To determine the acceleration of a point at a given moment in time, we calculate the second derivative of the equation of motion with respect to time

$$a(t) = \frac{dv}{\mathrm{dt}} = \frac{d}{\mathrm{dt}}(2t - 10) = 2\,\mathrm{m/s^2}\ . \qquad (4)$$

It is crucial to note that since the acceleration a is constant, the motion of the point particle is uniformly accelerated. Equations (3) and (4) reveal that the initial velocity and acceleration have opposite signs, indicating that the direction of velocity and acceleration at the initial time is opposite [6, 491]. A general physical analysis suggests that initially, the velocity will decrease in magnitude until it reaches a stopping point.

For further analysis of body movement, we will determine the time at which the point particle comes to a stop. This can be accomplished by identifying the moment when the speed of the point particle becomes zero

$$v(t) = 0\ . \qquad (5)$$

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Let's rewrite Equation (2) under the condition specified in (5) and determine the stopping time of the point particle

$$2t_{\text{stop}} - 10 = 0\,, \quad (6)$$

where t_{stop} represents the stopping time.

Solving Equation (6), we get

$$t_{\text{stop}} = 5\,\text{s}\,. \quad (7)$$

Thus, employing the analytical method, we determined that the point particle comes to a stop now $t_{\text{stop}} = 5$ s. This confirms the existence of a single stopping point during the movement of a point particle in the time interval from 1 to 8 seconds.

1.1. Calculating Point Movement Characteristics Using Wolfram Mathematica

When studying the physical parameters of the movement of a point particle, we recommend that students calculate these parameters both manually and using the Wolfram Mathematica symbolic calculation package. By combining traditional methods with modern computational tools, this approach enhances the understanding of physical concepts and fosters the development of skills in the analysis of physical phenomena. Here's a clearer and illustrative example of potential code in the Wolfram Language [7] for conducting the calculations mentioned:

```
x[t]=3+-10t+t^2;
t1=1;
t2=8;
Print["Task:"]
Print["Given the kinematic equation of motion x(t)=", x[t]]
Print[" t1 = ", t1," s;"]
Print[" t2 = ", t2," s;"]
Print["Solution:"]
Print["Let's find the velocity as a function of time
v(t) = dx/dt = ", D[x[t],t]]
v=D[x[t],t];
eq=Solve[D[x[t],t]==0,t];
tstop=t/.eq;
If[t1<tstop[[1]]<t2, tstop=tstop[[1]]];
Print["We find the stopping time from the equation ",
D[x[t],t],"=0. "]
Print["Thus we get  tstop = ", tstop, " s."]
a=D[v,t];
Print[" The acceleration as a function of time is
a(t)= ", a, " m/s2."]
```


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After executing this code, we obtain the values of the movement parameters, which are comparable to those obtained by the analytical solution and provided above (Fig. 1).

```
Task:
Given the kinematic equation of motion x(t)=3 - 10 t + t^2
 t1 = 1 s;
 t2 = 8 s;
Solution:
 Let's find the velocity as a function of time
v(t) = dx/dt = -10 + 2 t
We find the stopping time from the equation -10 + 2 t=0.
Thus we get  tstop = 5 s.
The number of stopping points = 1
 We will consider the stopping points for which t1 < tstop < t2
 The acceleration as a function of time is
a(t) = 2 m/s^2;
```

Fig. 1. Results of calculating motion parameters using Wolfram Mathematica

This approach not only facilitates the learning of the material but also sustains students' interest in physics, enabling them to comprehend and apply theoretical concepts in practice.

1.2. Graphical and Analytical Analysis of Motion Parameters Using Excel

For a visual representation of the relationship between point particle speed and time, we propose a comprehensive approach that combines analytical and graphical methods. In our work, we use Excel [8, 81–134] to visually depict the variation in point particle speed over time. This method allows for the visual representation of the graphical dependence of point particle speed on time and the identification of the moment when the point particle comes to a stop (Fig. 2), where the speed is calculated according to equation (2).

The next step is to determine the displacement and path of the point particle. The displacement is calculated using the formulas

$$\Delta x = x_2 - x_1 = x(t_2) - x(t_1)\,. \tag{8}$$

Numerical calculations yield the following result

$$\Delta x = -13 + 6 = -7\,\text{m}\,. \tag{9}$$

t, s	v, m/s
1	-8
1,5	-7
2	-6
2,5	-5
3	-4
3,5	-3
4	-2
4,5	-1
5	0
5,5	1
6	2
6,5	3
7	4
7,5	5
8	6

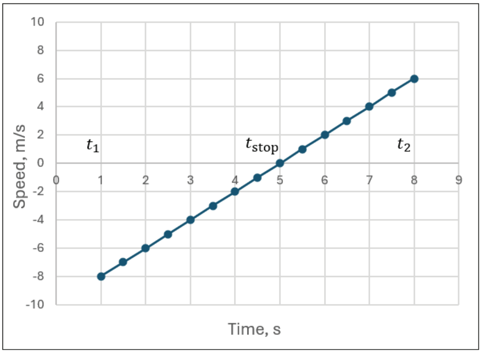

Fig. 2. Plot of the dependence of speed on time, $v(t)$

Calculating the path is a more complex task, and therefore, we suggest that students approach it in two ways: analytically and graphically.

1.3. Analytical Method for Path Determination

This method involves applying physical principles and formulas to determine the movement of a point particle. Since we are dealing with a one-dimensional problem, it is evident that the traveled path *S* during the time Δ*t* will be equal to the magnitude of displacement if the point particle moved in one direction

$$S(t, t+\Delta t) = |\Delta x| = |x(t+\Delta t) - x(t)|\,. \tag{10}$$

In previous calculations, it was determined that between the time $t_1 = 1$ s and $t_2 = 8$ s, the point particle stops at $t_{stop} = 5$ s. Thus, the complete path during this time can be calculated as the sum of two paths from t_1 to t_{stop} and from t_{stop} to t_2

$$S(t_1, t_2) = S(t_1, t_{stop}) + S(t_{stop}, t_2)\,. \tag{11}$$

Using expressions (11) and (10), we can write

$$S(t_1, t_2) = |\mathrm{x}(t_{stop}) - \mathrm{x}(t_1)| + |\mathrm{x}(t_2) - \mathrm{x}(t_{stop})|\,. \tag{12}$$

Then, substituting the numerical values into (12), we obtain

$$S(1,8) = |-22+6| + |-13+22| = 25\,\mathrm{m}. \tag{13}$$

1.4. Graphical Approach to Path Determination

The graphical method for path calculation is another tool that aids in determining the path of the body in practice [9]. To apply the graphical method, we use the data presented in Figure 2. From this graph, it can be

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observed that two right-angled triangles S_1 and S_2, can be distinguished, areas of which correspond to the distance traveled by the body before and after stopping. The area of a right triangle can be calculated using the formula

$$S = \frac{1}{2} a \cdot b \,, \tag{14}$$

where a and b are the legs of the right triangles.

Thus, using formula (14), we can determine the areas S_1 and S_2

$$S_1 = \frac{1}{2} \cdot \left| v(t_1) \right| \cdot \left(t_{stop} - t_1 \right),$$

$$S_2 = \frac{1}{2} \cdot v(t_2) \cdot \left(t_2 - t_{stop} \right). \tag{15}$$

After performing the numerical calculations, we obtain

$$S_1 = \frac{1}{2} \cdot 8 \cdot (5-1) = 16 \,\text{square units},$$

$$S_2 = \frac{1}{2} \cdot 6 \cdot (8-5) = 9 \,\text{square units}. \tag{16}$$

It is evident that in this case, the square units are equivalent to meters.

The total path is calculated as the sum of S_1 and S_2

$$S = S_1 + S_2 = 25\,\text{m} \,. \tag{17}$$

The obtained results confirm the effectiveness of both methods for calculating the path of point particle movement. The study demonstrates that both methods yield the same path value, as shown by formulas (13) and (17), confirming their equivalence and facilitating a deeper understanding of the physical laws of kinematics.

In our classes, we recommend that students utilize mathematical software, such as Wolfram Mathematica, to perform calculations related to the movement and path of a point particle. We provide an example of code that can be used to perform these calculations:

```
v1 = v /. t -> t1;
v2 = v /. t -> t2;
Print["The legs of the first triangle are:"]
Print["|V1| = ", Abs[v1], " units;"]
Print["(tstop - t1) = ", tstop - t1, " units."]
Print["The legs of the second triangle are:"]
Print["|V2| = ", Abs[v2], " units;"]
Print["(t2 - tstop) = ", t2 - tstop, " units."]
Print["The area of triangle S1 is S1 = |V1| * (tstop - t1)/2 = ",
Abs[v1], "*", tstop - t1, "/2 = ", Abs[v1] * (tstop - t1)/2, " units^2."]
```

```
Print["The area of triangle S2 is S2 = |V2| * (t2 - tstop)/2 = ",
Abs[v2], "*", t2 - tstop, "/2 = ", Abs[v2] * (t2 - tstop)/2, " units^2."]
Print["The total path is S = S1 + S2 = ",
Abs[v1] * (tstop - t1)/2, " + ", Abs[v2] * (t2 - tstop)/2, " = ",
Abs[v1] * (tstop - t1)/2 + Abs[v2] * (t2 - tstop)/2, " m."]
Print["The total displacement is \Delta x = S2 - S1 = ",
Abs[v2] * (t2 - tstop)/2, " - ", Abs[v1] * (tstop - t1)/2, " = ",
Abs[v2] * (t2 - tstop)/2 - Abs[v1] * (tstop - t1)/2, " m."]
```

The results of the code analysis are illustrated in Figure 3.

The legs of the first triangle are:

$|V_1| = 8$ units

$(tstop-t_1) = 4$ units

The legs of the second triangle are:

$|V_2| = 6$ units

$(t_2-tstop) = 3$ units

The area of triangle S_1 is $S_1 = |V_1| \times (tstop-t_1)/2 = 8\times4/2 = 16$ units^2

The area of triangle S_2 is $S_2 = |V_2| \times (t2-tstop)/2 = 6\times3/2 = 9$ units^2

The total path is

$S = S_1+S_2 = 16 + 9 = 25$ m

The total displacement is

$\Delta x = S_2-S_1 = 9 - 16 = -7$ m

Fig. 3. Results of Calculated Path and Displacement

2. Generating graphical representations for $S(t)$ and $x(t)$

Formula (1) was used to plot the graphical dependence of the coordinate on time within the interval from 1 to 8 seconds. Using this formula allowed for the numerical calculation of the body's coordinates at various time points. The results of these calculations are shown in the table below, providing a visual representation of the body's trajectory from 1 to 8 seconds (see Fig. 4).

Analyzing this graphical representation, we observe that the coordinate $x(t)$ initially decreases, reaching a minimum value at the stopping moment t_{stop} = 5 s, and then increases again. This indicates that the point first moves in one direction, comes to a stop, and then reverses its direction of motion.

Constructing the dependence $S(t)$ is a more complex task. From theoretical material, it is known that the path S is always a non-decreasing quantity during the movement of a body [10, 180]. Thus, the formula for

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t, s	x, m
1	-6
1,5	-9,75
2	-13
2,5	-15,75
3	-18
3,5	-19,75
4	-21
4,5	-21,75
5	-22
5,5	-21,75
6	-21
6,5	-19,75
7	-18
7,5	-15,75
8	-13

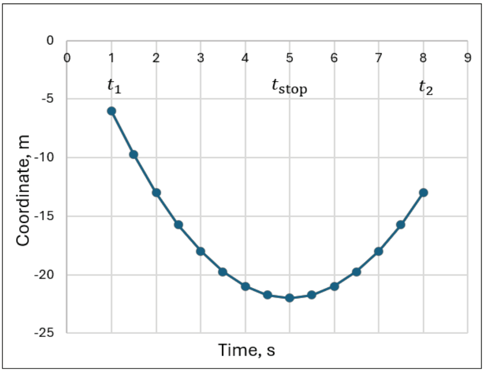

Fig. 4. Graph of the coordinate versus time, $x(t)$

calculating the path from t_1 to t_2, accounting for one stopping point, is given by

$$S(t)=\begin{cases} \left|x(t)-x(t_1)\right|, & t<t_{stop} \\ \left|x(t_{stop})-x(t_1)\right|+\left|x(t)-x(t_{stop})\right|, & t\geq t_{stop} \end{cases}. \quad (18)$$

To construct the graphical representation of $S(t)$, we will use the Excel program. The results of the calculations and the graphical representation of $S(t)$ are shown in Figure 5. The graphical representation of the path complements our analytical approach.

t, c	S, m
1	0
1,5	3,75
2	7
2,5	9,75
3	12
3,5	13,75
4	15
4,5	15,75
5	16
5,5	16,25
6	17
6,5	18,25
7	20
7,5	22,25
8	25

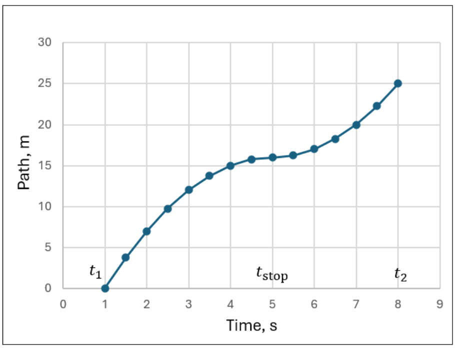

Fig. 5. Graph of the path versus time, $S(t)$

Analyzing the coordinate and path graphs, we observe that the distance traveled is always a positive quantity and continuously increases over time. This confirms that the distance traveled by the point from t_1 to t_2 consistently increases with time.

Conclusions. Based on the research outcomes, we can affirm the significant efficacy of integrating analytical and graphical methods in the study of steady-state body movement. Analytical methods, including mathematical formulas and theoretical calculations, promote the development of mathematical thinking and the ability to abstractly model physical phenomena. In contrast, graphical techniques, using software like Excel and Wolfram Mathematica, enable students to identify tangible correlations between numerical data and graphical representations. This visual approach allows them to observe and understand the complex processes involved in studying body movement. Such an approach not only facilitates a deeper exploration of various aspects of physical phenomena but also motivates students to pursue independent research projects.

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АНОТАЦІЯ

Вивчаючи кінематику руху точок, студенти часто стикаються з труднощами у розумінні основних понять «шлях» і «переміщення». Ці терміни, хоча і взаємопов'язані, мають різні значення. У цій статті запропоновано ефективний підхід до вивчення цих важливих понять через детальний аналіз одновимірної задачі. Для полегшення засвоєння навчального матеріалу використано два методи розв'язання: геометричний та аналітичний. Порівняння цих підходів дозволяє студентам усвідомити їхні особливості та вибрати найбільш ефективний метод для конкретних умов задачі.

Основна увага статті зосереджена на обчисленні різних параметрів руху, таких як швидкість, час зупинки, переміщення і шлях, за допомогою програмного середовища Wolfram Mathematica. Використання цього програмного забезпечення надає студентам доступ до потужних аналітичних інструментів, що дозволяє їм проводити складні розрахунки, аналізувати результати та моделювати різні сценарії руху. Студенти можуть легко змінювати параметри задачі, такі як початкова швидкість, прискорення або тривалість руху, і спостерігати за змінами в отриманих результатах. Цей процес дозволяє глибше зрозуміти фізичні явища.

Після проведення аналітичного дослідження, в якому студенти розраховують параметри руху, використання інструментів Excel для створення графіків робить навчальний процес більш інтерактивним і наочним. Завдяки можливостям Excel студенти можуть легко візуалізувати дані, створюючи графіки, які ілюструють зміни параметрів руху в залежності від часу. Цей підхід не лише спрощує засвоєння

складних концепцій, а й сприяє розвитку навичок роботи з сучасними технологіями для аналізу даних і моделювання фізичних процесів, забезпечуючи тим самим комплексне навчання, яке включає як теорію, так і практичні навички.

Таким чином, інтеграція комп'ютерних технологій у навчальний процес підвищує якість освіти, стимулює зацікавленість студентів та готує їх до вирішення реальних завдань у наукових і практичних дослідженнях.

Ключові слова: *кінематика, шлях, рух, точка зупинки, аналітичний і графічний метод.*

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